Math 1650 Beginning and Intermediate Algebra Review

1. Consider the line whose graph is determined by the equation: $y = \dfrac{4-x}{3}$.

   (a) Write the slope-intercept form of this line, $y = mx + b$.

   (b) Find the slope of the line.

   (c) Find the y-intercept of the line.

   (d) Find the x-intercept of the line.

   (e) Sketch a detailed graph of the equation: $y = \dfrac{4-x}{3}$.

2. Write the slope-intercept form of the line containing the points $(-1, 7)$ and $(2, 1)$.

3. (a) Factor completely: $2x^4 + 5x^3 - 3x^2$.

   (b) Solve for x: $2x^4 + 5x^3 = 3x^2$.

4. Solve for x: $4x^2 - 2x + 1 = 0$:

   (a) by completing the square.

   (b) by applying the quadratic formula.

5. Consider the parabola whose graph is determined by the equation: $y = 2(x+3)^2 - 8$.

   (a) Find the vertex of this parabola.

   (b) Find the $y-$intercept of this parabola.

   (c) Find the $x-$intercept(s) of this parabola.

   (d) Sketch a detailed graph of the equation: $y = 2(x+3)^2 - 8$.

6. Consider the parabola whose graph is determined by the equation: $y = -2x^2 + 4x + 1$.

   (a) Find the vertex of this parabola.

   (b) Find the $y-$intercept of this parabola.

   (c) Find the $x-$intercept(s) of this parabola.

   (d) Sketch a detailed graph of the equation: $y = -2x^2 + 4x + 1$.

7. Solve for x:

   (a) $\dfrac{x^2}{x-3} = \dfrac{x}{x-2} + \dfrac{7x-12}{x^2-5x+6}$

   (b) $x + \sqrt{3-2x} = 0$.

   (c) $|2x-1| - 3 \geq 0$

   (d) $|2x-1| - 3 < 0$

Math 1650 Beginning and Intermediate Algebra Review Solutions

1. (a) To put in the form $y = mx + b$, we expand and rearrange terms:

$$\begin{aligned} y &= \frac{4-x}{3} \\ &= \frac{4}{3} - \frac{x}{3} \\ &= -\frac{x}{3} + \frac{4}{3} \\ &= -\frac{1}{3}x + \frac{4}{3} \end{aligned}$$

Final answer: $\boxed{y = -\frac{1}{3}x + \frac{4}{3}}$

(b) Slope: $\boxed{m = -\frac{1}{3}}$

(c) To find the $y-$intercept, we set $x = 0$ and solve for y:

$$\begin{aligned} y &= -\frac{1}{3}x + \frac{4}{3} \\ y &= -\frac{1}{3} \cdot 0 + \frac{4}{3} \\ y &= \frac{4}{3} \end{aligned}$$

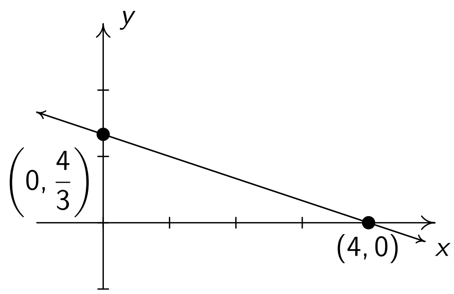
The $y-$intercept is: $\boxed{\left(0, \frac{4}{3}\right)}$

(d) To find the $x-$intercept, we set $y = 0$ and solve for x:

$$\begin{aligned} y &= -\frac{1}{3}x + \frac{4}{3} \\ 0 &= -\frac{1}{3}x + \frac{4}{3} \\ \frac{1}{3}x &= \frac{4}{3} \\ x &= 4 \end{aligned}$$

The $x-$intercept is: $\boxed{(4, 0)}$

1. (e) The graph of $y = \dfrac{4-x}{3}$:



2. We are given two points: $(x_1, y_1) = (-1, 7)$ and $(x_2, y_2) = (2, 1)$. We can find the slope, m as follows:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1-7}{2-(-1)} \\ &= -2. \end{aligned}$$

Next, we use the point-slope form of a line:

$$\begin{aligned} y &= m(x - x_1) + y_1 \\ &= -2(x - (-1)) + 7 \\ &= -2x + 5. \end{aligned}$$

Final answer: $\boxed{y = -2x + 5}$

3. (a) $2x^4 + 5x^3 - 3x^2 = x^2\left(2x^2 + 5x - 3\right) = x^2(2x-1)(x+3)$

   (b) Solving for x:

$$\begin{aligned} 2x^4 + 5x^3 &= 3x^2 \\ 2x^4 + 5x^3 - 3x^2 &= 0 \\ x^2(2x-1)(x+3) &= 0 \end{aligned}$$

Hence, either $x^2 = 0$, or $2x - 1 = 0$, or $x + 3 = 0$. Final answers: $\boxed{x = 0, \frac{1}{2}, -3}$

4. (a) Solving using Completing the Square:

$$\begin{aligned}
4x^2 - 2x + 1 &= 0 \\
4x^2 - 2x &= -1 \\
x^2 - \frac{1}{2}x &= -\frac{1}{4} \\
\left(x - \frac{1}{4}\right)^2 &= -\frac{1}{4} + \frac{1}{16} \\
\left(x - \frac{1}{4}\right)^2 &= -\frac{3}{16} \\
x - \frac{1}{4} &= \pm\sqrt{-\frac{3}{16}} \\
x - \frac{1}{4} &= \pm\frac{i\sqrt{3}}{4} \\
x &= \frac{1}{4} \pm \frac{i\sqrt{3}}{4}
\end{aligned}$$

Final answers: $\boxed{x = \frac{1 \pm i\sqrt{3}}{4}}$

(b) Solving $4x^2 + 2x + 1 = 0$ using the Quadratic Formula: $a = 4$, $b = -2$, $c = 1$:

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(1)}}{2(4)} \\
&= \frac{2 \pm \sqrt{-12}}{8} \\
&= \frac{2 \pm 2i\sqrt{3}}{8} \\
&= \frac{2\left(1 \pm i\sqrt{3}\right)}{8} \\
&= \frac{1 \pm i\sqrt{3}}{4}
\end{aligned}$$

As in part (a), we find our final answers: $\boxed{x = \frac{1 \pm i\sqrt{3}}{4}}$

5. (a) The equation $y = 2(x+3)^2 - 8$ is in standard form. Hence, the vertex is: $\boxed{(-3,-8)}$

(b) To find the $y-$intercept, we set $x = 0$ and solve for y:

$$\begin{aligned} y &= 2(x+3)^2 - 8 \\ &= 2(0+3)^2 - 8 \\ &= 10 \end{aligned}$$

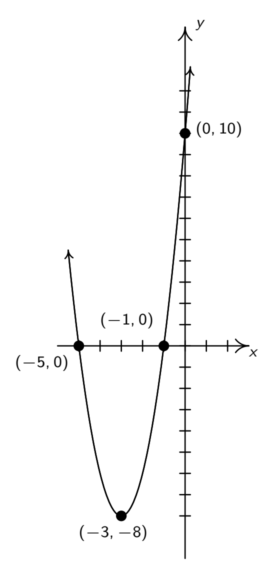
The $y-$intercept is: $\boxed{(0,10)}$

(c) To find the $x-$intercept, we set $y = 0$ and solve for x:

$$\begin{aligned} y &= 2(x+3)^2 - 8 \\ 0 &= 2(x+3)^2 - 8 \\ 8 &= 2(x+3)^2 \\ 4 &= (x+3)^2 \\ (x+3)^2 &= 4 \\ x+3 &= \pm 2 \\ x &= -3 \pm 2 \end{aligned}$$

Hence, $x = -3+2 = -1$ or $x = -3-2 = -5$. The $x-$intercepts are: $\boxed{(-5,0),\ (-1,0)}$

(d) The graph of $y = 2(x+3)^2 - 8$:



6. (a) The equation $y = -2x^2 + 4x + 1$ is not in standard form, so we use the vertex formula with $a = -2$ and $b = 4$:

$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{4}{2(-2)} \\ &= 1. \end{aligned}$$

To find the $y-$value of the vertex, we substitute $x = 1$:

$$\begin{aligned} y &= -2x^2 + 4x + 1 \\ &= -2(1)^2 + 4(1) + 1 \\ &= 3 \end{aligned}$$

The vertex is: $\boxed{(1, 3)}$

(b) To find the $y-$intercept, we set $x = 0$ and solve for y:

$$\begin{aligned} y &= -2x^2 + 4x + 1 \\ &= -2(0)^2 + 4(0) + 1 \\ &= 1 \end{aligned}$$

The $y-$intercept is $\boxed{(0, 1)}$

(c) To find the $x-$intercept, we set $y = 0$ and solve for x:

$$\begin{aligned} y &= -2x^2 + 4x + 1 \\ 0 &= -2x^2 + 4x + 1 \\ 2x^2 - 4x - 1 &= 0 \end{aligned}$$

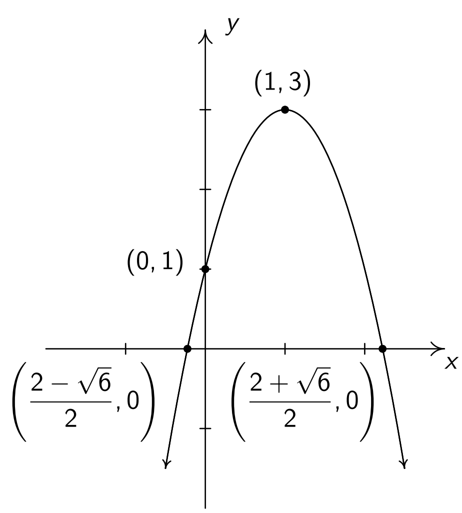
Since $2x^2 - 4x - 1$ doesn't factor nicely, we'll use the Quadratic Formula to solve this equation.

6. (c) (Continued.) Using the Quadratic formula: $a = 2$, $b = -4$, $c = -1$:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{4 \pm \sqrt{24}}{4} \\ &= \frac{4 \pm 2\sqrt{6}}{4} \\ &= \frac{2\left(2 \pm \sqrt{6}\right)}{4} \\ &= \frac{2 \pm \sqrt{6}}{2} \end{aligned}$$

The $x-$intercepts are: $\boxed{\left(\frac{2+\sqrt{6}}{2}, 0\right), \left(\frac{2-\sqrt{6}}{2}, 0\right)}$

(d) The graph of $y = -2x^2 + 4x + 1$:



7. (a) Solve for x:

$$\begin{aligned}
\frac{x^2}{x-3} &= \frac{x}{x-2} + \frac{7x-12}{x^2-5x+6} \\
\frac{x^2}{x-3} &= \frac{x}{x-2} + \frac{7x-12}{(x-2)(x-3)} \\
\left[\frac{x^2}{x-3}\right]\cdot(x-2)(x-3) &= \left[\frac{x}{x-2} + \frac{7x-12}{(x-2)(x-3)}\right]\cdot(x-2)(x-3) \\
x^2(x-2) &= x(x-3) + 7x - 12 \\
x^3 - 2x^2 &= x^2 - 3x + 7x - 12 \\
x^3 - 3x^2 - 4x + 12 &= 0 \\
x^2(x-3) - 4(x-3) &= 0 \\
\left(x^2-4\right)(x-3) &= 0 \\
(x-2)(x+2)(x-3) &= 0
\end{aligned}$$

Hence, $x-2=0$, or $x+2=0$, or $x-3=0$, so we have: $x=-2,2,3$. Since we multiplied both sides of the equation by a variable, we need to check for extraneous solutions. Substituting our solutions for x into the original equation shows that $\boxed{x=-2}$ is the only solution.

7. (b) Solve for x:

$$\begin{aligned} x+\sqrt{3-2x} &= 0 \\ x &= -\sqrt{3-2x} \\ x^2 &= \left(-\sqrt{3-2x}\right)^2 \\ x^2 &= 3-2x \\ x^2+2x-3 &= 0 \\ (x+3)(x-1) &= 0 \end{aligned}$$

Hence, $x+3=0$ or $x-1=0$, so we have $x=-3,1$. Since we raised both sides of the equation to an even power, we need to check for extraneous solutions. Substituting our solutions for x into the original equation shows that $\boxed{x=-3}$ is the only solution.

(c) Solve for x:

$$\begin{aligned} |2x-1|-3 &\geq 0 \\ |2x-1| &\geq 3 \\ 2x-1\geq 3 \quad &\text{or} \quad 2x-1\leq -3 \\ x\geq 2 \quad &\text{or} \quad x\leq -1 \end{aligned}$$

In interval notation, our answer is: $\boxed{(-\infty,-1]\cup[2,\infty)}$

(d) Solve for x:

$$\begin{aligned} |2x-1|-3 &< 0 \\ |2x-1| &< 3 \\ -3 < 2x-1 &< 3 \\ -2 < 2x &< 4 \\ -1 < x &< 2 \end{aligned}$$

In interval notation, our answer is: $\boxed{(-1,2)}$